

# Conceptual unification of elementary particles, black holes, quantum de Sitter and Anti de Sitter string states

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We provide a conceptual unified description of the quantum properties of black holes (BH), elementary particles, de Sitter (dS) and Anti de Sitter (AdS) string states. The conducting line of argument is the classical-quantum (de Broglie, Compton) duality here extended to the quantum gravity (string) regime (wave-particle-string duality). The semiclassical (QFT) and quantum (string) gravity regimes are respectively characterized and related: sizes, masses, accelerations and temperatures. The Hawking temperature, elementary particle and string temperatures are shown to be the same concept in different energy regimes and turn out the precise classical-quantum duals of each other; similarly, this result holds for the BH decay rate, heavy particle and string decay rates; BH evaporation ends as quantum string decay into pure (non mixed) radiation. Microscopic density of states and entropies in the two (semiclassical and quantum) gravity regimes are derived and related, an unifying formula for BH, dS and AdS states is provided in the two regimes. A string phase transition towards the dS string temperature (which is shown to be the precise quantum dual of the semiclassical (Hawking-Gibbons) dS temperature) is found and characterized; such phase transition does not occurs in AdS alone. High string masses (temperatures) show a further (square root temperature behaviour) sector in AdS. From the string mass spectrum and string density of states in curved backgrounds, quantum properties of the backgrounds themselves are extracted and the quantum mass spectrum of BH, dS and AdS radii obtained.

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## I. INTRODUCTION AND RESULTS

Macroscopic black holes arise through the gravitational collapse of stellar bodies. Microscopic black holes could arise from high density concentrations (of the order of the Planck energy scale) in the early universe, as well as from the collisions of particles at such energy scales. Microscopic black holes are necessarily quantum and their properties governed by quantum or semiclassical gravity, evaporation through Hawking radiation is a typical effect of these black holes. Microscopic black holes share in some respects analogies with elementary particles, and on the other hand, show many important differences. A theory of quantum gravity, or “theory of everything” such as string theory, should accounts for an unified and consistent description of both black holes and elementary particles, and the physics of the early universe as well.

In this paper, we provide an unifying description of the quantum properties of black holes, elementary particles, de Sitter and Anti-de Sitter states. The conducting line of argument is the concept of classical-quantum (wave-particle) duality at the basis of quantum mechanics, here extended to include the quantum gravity string domain, ie wave-particle-string duality. We set up the relevant scales characteristic of the semiclassical gravity regime (QFT matter + classical gravity) and relate it to the classical and quantum gravity regimes. The de Broglie-Compton wave length  $L_q = \hbar(M_{cl}c)^{-1}$  in the presence of gravity means  $L_q = l_{Pl}^2 L_{cl}^{-1}$  (“Planck” duality),  $L_{cl}$  and  $l_{Pl}$  being the gravitational ( $L_{cl} = \frac{G}{c^2} M_{cl}$ ) and Planck length scales respectively. As quantum behaviour is the dual of classical behaviour (through  $\hbar, c$ ), the dual of semiclassical gravity regime is full quantum gravity regime (through  $\hbar, G, c$ ), (in string theory through  $\hbar, \alpha', c$ ). The semiclassical gravity mass scale is the “dual” mass  $M_{sem} = m_{Pl}^2 M_{cl}^{-1}$ , its corresponding temperature scale (energy) is the Hawking temperature  $T_{sem} = \frac{1}{2\pi k_B} M_{sem} c^2$ , ( $m_{Pl} \equiv$  Planck mass).

The set of quantities  $O_{cl, sem} = (L_{cl}, M_{cl}, \mathcal{K}_{cl}, T_{sem})$  characteristic of the classical/semiclassical gravity regime, (here denoting size, mass, gravity acceleration, Hawking temperature respectively), and the corresponding set in the full quantum gravity regime  $O_q = (L_q, M_q, \mathcal{K}_q, T_q)$  are **dual** (in the sense of the wave-particle duality) of each other, ie

$$O_{cl, sem} = o_{Pl}^2 O_q^{-1} \quad (1.1)$$

This relation holding for each quantity in the set.  $O_{cl, sem}$  and  $O_q$  being the same conceptual physical quantities in the different (classical/semiclassical and quantum) gravity regimes. ( $o_{Pl}$  purely depending of  $(\hbar, c, G)$ ).  $O_q$  standing for the usual concepts of quantum size, mass, acceleration and temperature (ie  $T_q = \frac{1}{2\pi k_B} M c^2$ ). In string theory we have similarly,

$$O_{cl, sem} = o_s^2 O_s^{-1} \quad (1.2)$$

with  $\alpha'$  instead of  $G/c^2$  (and  $o_s^2$  purely depending on  $(\hbar, \alpha', c)$ ).  $O_s = (L_s, M_s, \mathcal{K}_s, T_s)$  denoting here the characteristic string size, string mass, string acceleration and string temperature of the system under consideration, ( $T_s = \frac{1}{2\pi k_B} M_s c^2$ ), which are in general different from the usual flat space string expressions, (in particular they can be equal). This is not an assumed or conjectured duality: the results of QFT and string quantization in black holes, de Sitter, Anti-de Sitter and WZWN backgrounds remarkably show these relations [1],[2],[3] and sections below. As the wave-particle duality, the semiclassical-quantum gravity duality does not relate to the number of dimensions, nor to a particular symmetry. In this paper we also extend and use these relations to include the microscopic density of states and entropies ( $\rho_{sem}, S_{sem}$ ) and ( $\rho_s, S_s$ ), as well. The gravitational size  $L_{cl}$  in the classical regime becomes the quantum size  $L_q$  or string size  $L_s$  in the quantum gravity regime. The QFT Hawking temperature  $T_{sem}$  in the semiclassical gravity regime becomes the string temperature  $T_s$  in the full quantum gravity regime. We find ( $\rho_{sem}, S_{sem}$ ) in the semiclassical gravity regime such that they become ( $\rho_s, S_s$ ) in the quantum gravity string regime. The asymptotic (high M) expressions are given by

$$e^{S_{sem}/k_B} = \rho_{sem}(M_{cl}) = \left( \frac{S_{sem}^{(0)}}{k_B} \right)^{-a} e^{S_{sem}^{(0)}/k_B}, \quad \text{where} \quad (1.3)$$

$$S_{sem}^{(0)} = \frac{k_B}{4} \frac{A}{l_{Pl}^2} = \frac{\pi k_B}{p} \frac{T(M_{cl})}{T_{sem}} = \frac{1}{2p} \frac{M_{cl} c^2}{T_{sem}}$$

(here  $a = D$  space time dimensions, and with our normalization of  $L_{cl} = \frac{G}{c^2} M_{cl}$  :  $p = 4$  for BH's,  $p = 1$  for dS and AdS). The leading term  $S_{sem}^{(0)}$  is the known gravitational entropy ( $A \equiv$  horizon area), (in the absence of event horizon, as in AdS,  $A$  is the area associated to  $L_{cl}$ , as  $T_{sem}$  just becomes a temperature scale associated to  $M_{sem}$ , the Hawking temperature in AdS being formally zero).

The string density of mass states and string entropy in *BH* (asymptotically flat) backgrounds are the same as in flat space time [4]. We find  $\rho_s$  and  $S_s$  in dS and AdS backgrounds which are different from their flat space expressions (the

string mass spectra in dS and AdS are different from the flat space expressions) [5],[6],[7]). The asymptotic formulae (high M) are

$$e^{S_s/k_B} = \rho_s(M) = f(M) \left( \frac{S_s^{(0)}}{k_B} \right)^{-a} e^{S_s^{(0)}/k_B} \quad (1.4)$$

$$S_s^{(0)}(M) = \frac{1}{2\pi} \frac{Mc^2}{T_S}, \quad T_S = \frac{1}{2\pi k_B} M_S c^2$$

where,

$$\begin{aligned} p=4, \quad M_S &= \frac{1}{8b} \sqrt{\frac{\hbar}{\alpha' c}}, \quad f=1: \quad BH \\ p=1, \quad M_S &= \frac{1}{8b} \frac{c}{\alpha' H}, \quad f = \sqrt{\frac{M_S}{M_S - M}}: \quad dS \\ p=1, \quad M_S &= \frac{1}{8b} \frac{c}{\alpha' H}, \quad f=1: \quad AdS. \end{aligned} \quad (1.5)$$

Here H is the Hubble constant and  $b = 2\sqrt{\frac{D-2}{6}}$ , (D = number of space-time dimensions). In BH's and AdS backgrounds, high string masses are not bounded, while in dS space-time  $M_S$  is a maximal mass for the oscillating (particle) string states. Remarkably, for  $M \rightarrow M_S$ , the string density of states and entropy ( $\rho_S, S_S$ ) indicate the presence of a phase transition at the de Sitter string temperature  $T_S$ ; the square root branch behaviour near  $T_S$  is universal: it holds in any number of dimensions, and is analogous to that found in the thermal self-gravitating gas of (non-relativistic) particles (by mean field and Monte Carlo methods) [8]. In AdS background alone (including WZWN models) such phase transition does not occur, (there is no finite (at finite temperature) critical point: (the negative  $\Lambda$  (curvature) pushes it to infinity ; ( $\rho_S, S_S$ ), the partition function are all finite in AdS). (The Hawking-Page transition [9] which occurs in the context of semiclassical BH-AdS systems is due to the BH, not to AdS which acts only as a space boundary condition). In AdS backgrounds (including WZWN models) the very high string masses ( $M \gg M_S$ ) show a new sector (non existing in BH's nor in dS background):

$$S_S^{(0)}(M \gg M_S) = \pi k_B \frac{\sqrt{MM_S}}{m_S} = \frac{\sqrt{TT_S}}{t_S}$$

The characteristic mass ratio (or temperature) in  $\rho_S(M)$  and  $S_S(M)$  from the low to the high masses follows the behaviour

$$(i) \frac{M}{m_S} (M \ll M_S) \rightarrow (ii) \frac{M_S}{m_S} (M \approx M_S) \rightarrow (iii) \frac{\sqrt{MM_S}}{m_S} (M \gg M_S),$$

which, with our dual relation  $\frac{t_S}{T_S} = \frac{T_{sem}}{t_S}$ , reads

$$(i) \frac{T}{t_S} (TT_{sem} \ll t_S^2) \rightarrow (ii) \frac{t_S}{T_{sem}} (TT_{sem} \approx t_S^2) \rightarrow (iii) \sqrt{\frac{T}{T_{sem}}} (TT_{sem} \gg t_S^2)$$

The sector (iii) is absent in BH's and dS backgrounds. Interestingly enough, the highly excited string spectrum and our classical-quantum gravity dual relations allow to infer quantum properties of the background itself. The mass scale for the low masses is the fundamental string mass  $m_S = \sqrt{\hbar/c\alpha'}$  while for the high masses the scale is  $M_S$ . For  $m \rightarrow M_S$ , the string becomes the background (and conversely, the background becomes the string): it turns out that  $M_S$  is the mass  $M_{cl}$  of the background for  $(G/c^2) \rightarrow \alpha'$ , and conversely (ie  $M_{cl} \Leftrightarrow M_S$  with  $(G/c^2) \Leftrightarrow \alpha'$ ). The massive string properties reproduce those of the background, and the quantum mass spectrum of the background is obtained such that it becomes the highly massive string spectrum in the quantum gravity regime. We obtain

$$M_{cl} = \left( \frac{2\pi k_B}{c^2} T_{sem} \right) 2pb^2 N, \quad ie \quad M_{cl} = m_{mpl} 2pb\sqrt{N}, \quad (b = 2\sqrt{\frac{D-2}{6}})$$

where p=4 for BH's, p=1 for dS and AdS. This spectrum hold in any number of dimensions. This is like the high N mass spectrum of strings in BH's, dS and AdS backgrounds when M tends to  $M_S$ . For the BH and dS radii (and AdS scale length) we have

$$R_{BH} = \frac{(D-3)}{2} l_{pl} 8b\sqrt{N}, \quad \frac{c}{H} = 2bl_{pl}\sqrt{N}$$

Quantum string properties of BH's, dS and AdS states are determined by  $(L_S, M_S, K_S, T_S)$  (size, mass, acceleration and temperature of strings in these backgrounds), and their quantum spectra determined by the highly excited string spectra. These string phase properties confirm and complete the back reaction results found in the string analogue model (thermodynamical approach)[1],[2]). The QFT semiclassical BH's, dS and AdS backgrounds are a low curvature (low energy) phase (for  $T_{sem} \ll T_S$ ) of the string phase reached when  $T_{sem} \rightarrow T_S$ , (and so when  $L_{cl} \rightarrow L_S, K_{cl} \rightarrow K_S$ ). The two phases are dual (in the sense of the wave-particle-string duality) of each other. From these findings, the last phase of black hole evaporation is shown to follow the same decay formula as heavy elementary particle decay or string decay. This completes in precise way the BH evaporation picture computed in ref[2]: string emission by BH's is an incomplete gamma function of  $(T_S - T_{sem})$  which for  $T_{sem} \ll T_S$  yields the QFT Hawking emission, and for  $T_{sem} \rightarrow T_S$  undergoes (back reaction) a phase transition to a string state which decays (as a string) into (most massless) particles including the graviton. As shown here, BH evaporation evolves precisely from a semiclassical gravity QFT phase (Hawking emission) with decay rate  $\Gamma_{sem} = \left| \dot{M}/M \right| \approx GT_{sem}^3$  to a quantum gravity string phase decaying as pure (non mixed) radiation with decay rate  $\Gamma_S = GT_S^3$ . Conversely, cosmological evolution goes from a quantum string phase (selfsustained by strings with  $T_S \gg T_{sem}$ ) to a semiclassical QFT phase (QFT inflation) and then to the classical (standard FRW) phase. The wave-particle-string duality precisely manifests in this evolution, between the different gravity regimes, and can be view as a mapping between asymptotic (in and out) states characterized by the sets  $O_{cl,sem}$  and  $O_S$ , and so as a S-matrix description.

This paper is organized as follows: in Section II we elaborate on the concept of wave-particle-string duality. Section III deals with semiclassical (QFT) and quantum (string) BH's: their microscopic density of states, entropies, quantum BH spectrum and decay. In sections IV and V we treat similar issues for the semiclassical and quantum dS and AdS states respectively, derived from the mass spectrum, density of string states and entropies. Some concluding remarks are given in section VI.

## II. WAVE-PARTICLE-STRING DUALITY

### A. Classical-quantum duality

In classical physics in the absence of gravity, there is no relation between mass and length (there is no G), and there is no temperature associated to the length. From a classical length  $L_{cl}$  one constructs an "acceleration"  $\mathcal{K}_{cl}$  (through c), and to a mass  $M_{cl}$  one associates a temperature (through Boltzmann constant  $k_B$ ) :

$$L_{cl} \xrightarrow{c} \mathcal{K}_{cl} = \frac{c^2}{L_{cl}} \quad , \quad T_{cl} = \frac{1}{2\pi k_B} M_{cl} c^2. \quad (2.1)$$

In quantum physics (QFT), mass and length are related through  $\hbar$  by the Compton length  $L_q$ . One constructs a (quantum) "acceleration"  $\mathcal{K}_q$  (through c), and an associated (quantum) temperature  $T_q$  (through  $k_B$ ) :

$$\lambda_q \equiv L_q = \frac{\hbar}{mc}, \quad \mathcal{K}_q = \frac{c^2}{L_q}, \quad T_q = \frac{1}{2\pi k_B} mc^2, \quad (2.2)$$

which can be also expressed as

$$\mathcal{K}_q = \frac{c^3}{\hbar} m, \quad T_q = \frac{\hbar c}{2\pi k_B} L_q^{-1} = \frac{\hbar}{2\pi k_B c} \mathcal{K}_q. \quad (2.3)$$

In the presence of gravity, length and mass are related (through G), the classical line eq (2.1) is written as :

$$L_{cl} = \frac{G}{c^2} M_{cl}, \quad \mathcal{K}_{cl} = \frac{c^4}{G} M_{cl}^{-1}, \quad T_{cl} = \frac{1}{2\pi k_B} \frac{c^4}{G} L_{cl} \quad (2.4)$$

The quantum line eq (2.2) is thus related to  $L_{cl}$  through the Planck length  $\sqrt{\hbar G/c^3} = l_{Pl}$  :

$$L_q = \frac{l_{Pl}^2}{L_{cl}}, \quad \mathcal{K}_q = \frac{c^2}{l_{Pl}^2} L_{cl}, \quad T_q = \frac{\hbar c}{2\pi k_B} \frac{1}{l_{Pl}^2} L_{cl} = T_{cl} \quad (2.5)$$

which also reads

$$L_q = \left( \frac{l_{Pl}}{c} \right)^2 \mathcal{K}_{cl}^{-1}, \quad \mathcal{K}_q = \left( \frac{c^2}{l_{Pl}} \right)^2 \mathcal{K}_{cl}^{-1}, \quad T_q = \frac{\hbar}{2\pi k_B} \left( \frac{c^2}{l_{Pl}} \right)^2 \mathcal{K}_{cl}^{-1} \quad (2.6)$$

(in terms of the classical “acceleration”  $\mathcal{K}_{cl}$ ). Classical and quantum *lengths* are inversely related to each other through  $G$ ,  $\hbar$  and  $c$ . (Without  $G$ , such relation does not exist :  $\hbar$  alone relates mass and quantum length, but not classical and quantum lengths).

### B. Semiclassical-quantum gravity duality (QFT)

In the context of QFT plus classical gravity, (semiclassical gravity), quantum matter is characterized by the relations eqs (2.2)-(2.3), classical gravity by the relations eq (2.4), and the combination of both gives rise to a non zero *semiclassical* temperature, which in the presence of event horizons is the Hawking temperature [10] :

$$T_{sem} = \frac{\hbar}{2\pi k_B c} \mathcal{K}_{cl} \quad (2.7)$$

$\mathcal{K}_{cl}$  is the classical acceleration eq.(2.1),(or surface gravity of the horizon) which in the black hole case is

$$\mathcal{K}_{cl} = \frac{c^2}{L_{cl}}, \quad L_{cl} = \frac{2}{D-3} R_{BH} \quad (2.8)$$

( $D$  being the number of space time dimensions),  $R_{BH}$  is the horizon radius. Therefore, the Hawking temperature which is a *semiclassical* concept for the black hole, can be expressed as

$$T_{sem} = \frac{\hbar c}{2\pi k_B} L_{cl}^{-1} = \frac{\hbar c}{2\pi k_B} \frac{L_q}{l_{Pl}^2} = \frac{1}{2\pi k_B} m_{Pl}^2 c^2 M_{cl}^{-1} \quad (2.9)$$

and we also write

$$T_{sem} = \frac{1}{2\pi k_B} M_{sem} c^2, \quad M_{sem} = \frac{m_{Pl}^2}{M_{cl}} \quad (2.10)$$

These expressions in terms of  $(L_{cl}, K_{cl}, M_{cl})$  or  $(L_q, K_q, M_{sem})$  hold in any number of space-time dimensions,  $D$  enters in eq (2.8) and in the relation between  $R_{BH}$  and the black hole mass  $M_{BH}$  :

$$R_{BH} = \left( \frac{16\pi G M_{BH}}{c^2(D-2)A_{D-2}} \right)^{1/(D-3)}, \quad \left( A_{D-2} \equiv \frac{2\pi^{(D-1)/2}}{\Gamma(\frac{D-1}{2})} \right) \quad (2.11)$$

and so for the black hole

$$M_{cl} = \frac{2}{(D-3)} \frac{c^2}{G} R_{BH} = \frac{2}{(D-3)} \frac{c^2}{G} \left( \frac{16\pi G M_{BH}}{c^2(D-2)A_{D-2}} \right)^{1/D-3} \quad (2.12)$$

The Hawking temperature is a measure of the Compton length of the black hole, and thus of its quantum properties in the *semiclassical* (or QFT) regime, that is when the Compton length  $L_q$  of the black hole is  $l_{Pl} \ll L_q \ll L_{cl}$ . Planck mass and Planck length satisfy by definition  $l_{Pl} = \frac{\hbar}{c} m_{Pl}^{-1}$  and  $l_{Pl} = \left( \frac{G}{c^2} \right) m_{Pl}$ . Classical and quantum black hole domains are related through the Planck scale :

$$L_{cl} L_q = l_{Pl}^2, \quad M_{cl} M_{sem} = m_{Pl}^2, \quad \mathcal{K}_{cl} \mathcal{K}_q = \kappa_{Pl}^2, \quad T_{sem} T_q = t_{Pl}^2 \quad (2.13)$$

$$(l_{Pl}^2 = \hbar G / c^3, \quad m_{Pl}^2 = \hbar c / G, \quad \kappa_{Pl} \equiv \frac{c^2}{l_{Pl}}, \quad t_{Pl} \equiv \frac{1}{2\pi k_B} m_{Pl} c^2) \quad (2.14)$$

$\kappa_{Pl}$  and  $t_{Pl}$  being the acceleration (or “surface gravity”) and temperature at the Planck scale respectively.  $L_{cl}, \mathcal{K}_{cl}, L_q, \mathcal{K}_q$  are given by eqs (2.4), (2.5) respectively. Expressions (2.4), (2.5), (2.7) also yield

$$L_{cl} = \left( \frac{M}{m_{Pl}} \right)^2 L_q, \quad \mathcal{K}_{cl} = \left( \frac{m_{Pl}}{M} \right)^2 \mathcal{K}_q, \quad T_{sem} = \left( \frac{m_{Pl}}{M} \right)^2 T_q, \quad (2.15)$$

showing the change of regime through the Planck mass domain. The tension (mass/length) being  $c^2/G$ .

It must be noticed that eqs (2.13) not only hold for black holes, but more generally they relate the semiclassical and quantum regimes of gravity. The semiclassical mass scale,  $M_{sem}$  eq. (2.10) we have introduced and its associated temperature scale eq. (2.10) are the characteristic scales of the semiclassical gravity regime. (In particular, in the presence of event horizons,  $\mathcal{K}_{cl}$  is the surface gravity of the horizon and  $T_{sem}$  is the Hawking temperature).

### C. Semiclassical (QFT)-string quantum gravity duality.

Quantum gravity string theory is naturally valid at the Planck scale. The Compton length of a quantum string is equal to its size

$$l_s = L_q = \frac{\hbar}{m_s c}, \quad (2.16)$$

and it also satisfies the “gravitational” (length-mass) relation

$$l_s = \alpha' m_s \quad (2.17)$$

The fundamental string constant  $\alpha'$  allows  $l_s$  being *both* proportional and inversely proportional to  $m_s$  : ( $\alpha'$  playing the role of  $[\frac{G}{c^2}]$ ). In general, the two lengths,  $L_q$  and  $l_s$  differ by a dimensionless number (string excitation), which we take here equal one (the ground state. All the relations eqs (2.2), (2.3) for elementary particles (QFT) hold universally for quantum strings (purely quantum objects), ( $q \equiv s$ ) :

$$l_s = \frac{\hbar}{m_s c} \quad , \quad \kappa_s = \frac{c^2}{l_s} \quad , \quad t_s = \frac{\hbar c}{2\pi k_B} \frac{1}{l_s} \quad (2.18)$$

$\kappa_s, t_s$  being respectively the fundamental string acceleration and string temperature

$$\kappa_s = \frac{c^2}{\alpha' m_s} = \frac{c^3}{\hbar} m_s, \quad t_s = \frac{\hbar}{2\pi k_B c} \kappa_s = \frac{1}{2\pi k_B} m_s c^2 \quad (2.19)$$

$$(l_s = \sqrt{\frac{\hbar \alpha'}{c}} \quad , \quad m_s = \sqrt{\frac{\hbar}{c \alpha'}} \quad , \quad t_s = \frac{\hbar c}{2\pi k_B \alpha'} m_s^{-1}) \quad (2.20)$$

Eqs (2.15)-(2.16) can be expressed entirely in terms of  $\alpha'$  (and  $\hbar, c$ ), (instead of  $l_{Pl}^2$ ),

$$L_{cl} L_s = l_s^2, \quad M_{sem} M_s = m_s^2, \quad \mathcal{K}_{cl} \mathcal{K}_s = \kappa_s^2, \quad T_{sem} T_s = t_s^2 \quad (2.21)$$

ie :

$$L_s = \frac{\hbar \alpha'}{c} L_{cl}^{-1}, \quad M_s = \frac{\hbar}{c \alpha'} M_{sem}^{-1} \quad , \quad \mathcal{K}_s = \frac{c}{\hbar \alpha'} c^4 \mathcal{K}_{cl}^{-1}, \quad T_s = \left( \frac{c}{2\pi k_B} \right)^2 \frac{\hbar c}{\alpha'} T_{sem}^{-1} \quad (2.22)$$

Eqs (2.7)-(2.10) are the semiclassical expressions of eqs (2.21)-(2.22). In the quantum string regime, the Hawking temperature  $T_{sem}$  becomes the string temperature  $T_s$  (the black hole becomes a string)[2],[3]. Eqs (2.21)-(2.22) are the quantum string analogue of eqs (2.7)-(2.10). The set  $(L_s, M_s, \mathcal{K}_s, T_s)$  is the **quantum string dual** (in the sense of the wave-particle string duality) of the classical/semiclassical (QFT) set  $(L_{cl}, M_{sem}, \mathcal{K}_{cl}, T_{sem})$ .

Eqs (2.20)-(2.21) are at the basis of the  $\mathcal{R}$  transform introduced in refs[1]-[3] in the context of Black holes and de Sitter space. Eqs (2.7)-(2.10) and (2.21)-(2.22) can be mapped one into another by a duality transform  $\mathcal{R}$

$$\mathcal{R}[O_{cl}] = O_q = o_s^2 O_{cl}^{-1} \quad \text{or} \quad \mathcal{R}[O_{sem}] = O_s = o_s^2 O_{sem}^{-1} \quad (2.23)$$

$O$  being the same physical magnitude in the different classical, semiclassical and quantum or string regimes,  $o_s$  purely depending on  $\alpha'$  (and  $c, \hbar$ ) (alternatively,  $o_{Pl}$  purely depending on the Planck length :  $G, c, \hbar$ ). An example of this  $\mathcal{R}$  transform is the mapping of lengths  $L_{cl}$  into  $L_q$  (and all their related magnitudes), eqs (2.20)-(2.22).

The dual of classical mechanics is quantum mechanics : a (wave) length is connected to the (inverse of) particle momentum through  $\hbar$  (de Broglie wave length or “de Broglie duality”). Its relativistic version (QFT), is the Compton length (“Compton duality”). In the presence of gravity, quantum and classical lengths are inversely connected to each other through the Planck length (“Planck duality”): semiclassical and quantum gravity regimes are dual of each other through the Planck scale domain. String theory being the consistent framework for quantum gravity, eqs (2.20)-(2.22) correspond to the quantum gravity string regime. As the dual of classical behaviour is quantum behaviour (with  $\hbar, c$ ), the dual of semiclassical (QFT) gravity is full quantum (string) gravity (through  $\alpha', \hbar, c$ ). This duality is *universal*. It is not linked to any symmetry, nor to the number or the kind of dimensions.

The size of the black hole is the gravitational length  $L_{cl}$  in the classical regime, it is the Compton length  $L_q$  in the semiclassical regime, it is the string size  $L_s$  in the full quantum gravity regime. Similarly, the horizon acceleration (surface gravity)  $\mathcal{K}_{cl}$  of the black hole is the string acceleration  $\mathcal{K}_s$  in the string regime. The Hawking temperature  $T_{sem}$  (measure of the surface gravity or of the Compton length) in the semiclassical gravity regime becomes the string temperature  $T_s$  in the full quantum gravity (string) regime. Moreover, (see section 3 below), the mass spectrum of the black hole in the semiclassical regime becomes the string spectrum in the full quantum regime.

This duality does not need *a priori* any symmetry nor compactified dimensions. It does not require the existence of any isometry in the curved background. Different types of relativistic quantum type operations  $L \rightarrow L^{-1}$  appear in string theory due to the existence of the dimensional string constant  $\alpha'$  (the most known being T-duality [11],[12],) linking physically equivalent string theories. The duality here we are considering is of the type classical-quantum (or wave-particle) duality (de Broglie “duality”), relating classical/semiclassical and quantum behaviours, here extended to include the quantum gravity string regime.

The de Broglie and Compton wave-lengths  $L_q = \hbar p^{-1}$  are not the expression of a symmetry transform between physically equivalent theories, but the crossing or the relationship between two different (classical and quantum) behaviours of Nature. The presence of  $G$  (or of  $\alpha'$ ) yields for the Compton length  $L_q = l_{Pl}^2 L_{cl}^{-1}$ , linking two different (semiclassical and quantum) gravity behaviours. This duality relation is supported explicitly from the results of QFT and quantum strings in curved backgrounds [1],[2],[3] and sections below. This is *not* an *assumed* or *conjectured* relation. The wave-particle-string duality, as the wave-particle duality at the basis of quantum mechanics, is reflected in the QFT and quantum string dynamics. As we will see below all relevant cases : black holes, de Sitter, AdS and WZWN-AdS backgrounds satisfy this QFT/string relation. The corresponding mass spectra and entropies as well (as we show in the sections below).

### III. SEMICLASSICAL (QFT) AND QUANTUM (STRING) BLACK HOLES

Difficulties in connecting black holes to elementary particles appear when comparing the known black hole properties (which are semiclassical and thermal) to the elementary particle properties (which are quantum and intrinsically non-thermal). Even when comparing black holes and strings, most frequently there are the *semiclassical* black hole properties those compared to, or derived from, strings [13], [14]. Semiclassical features (as Bekenstein-Hawking entropy) can be derived in string theory, but the real issue here, comparison and unification of black holes and elementary particles, makes sense only for *quantum* black holes, and it takes place at the quantum gravity scale.

The Hawking temperature is inversely proportional to the mass, while for an elementary particle, temperature is equivalent (through units conversion) to the mass (energy). Thermal intrinsic properties of black holes as known till now from the QFT (or string) derivation are *semiclassical*. Microscopic black holes are *quantum* objects and the semiclassical treatment is not enough to disclose their connection to quantum elementary particles. Recall that the thermal black hole properties *are already present* (formally) at the *classical* level (the four laws of black hole mechanics)[15]. Black hole thermodynamics can be compared to the thermodynamics of self gravitating (collapsed and relativistic) systems. In the quantum regime, quantum black holes become quantum strings, and the intrinsic thermal features of black holes are the intrinsic thermal features of strings. Quantum black holes at the Planck energy scale are just (become) particle (string) states. And/or, particle states at the Planck energy scale become quantum black holes.

#### A. Density of states and entropy

The black hole density of mass states  $\rho(M)$  in the semiclassical (QFT) regime satisfy :

$$\rho_{sem}(M) = e^{S_{sem}/k_B} \sim e^{M_{cl}c^2/(8k_B T_{sem})} \quad (3.1)$$

where

$$S_{sem} = \frac{k_B c^3}{G \hbar} \frac{A_{BH}}{4} = \frac{\pi k_B}{4} \left( \frac{L_{cl}}{l_{Pl}} \right)^2, \quad (3.2)$$

or,

$$S_{sem} = \frac{\pi k_B}{4} \left( \frac{M_{cl}}{m_{Pl}} \right)^2 = \frac{1}{8} \frac{M_{cl} c^2}{T_{sem}} = \frac{\pi k_B}{4} \frac{T_{cl}(M_{cl})}{T_{sem}} \quad (3.3)$$

$S_{sem}$  is the black hole entropy,  $T_{cl}$  is the classical temperature eq (2.1) and  $T_{sem}$  is the Hawking temperature eq (2.7) or (eqs (2.9), (2.10)).  $L_{cl}$  and  $M_{cl}$  are related to the black hole radius and mass by eqs (2.8) and (2.12) respectively (for  $D=4$ :  $L_{cl} = 2R_{BH}$ ,  $M_{cl} = 4M_{BH}$ ). The last two expressions in eq (3.3) are useful to compare with the corresponding string quantities.  $S_{sem}$  can be also expressed as

$$S_{sem} = \frac{\pi k_B}{4} \left( \frac{M_{cl}}{M_{sem}} \right) = \frac{\pi k_B}{4} \left( \frac{L_{cl}}{L_q} \right) \quad (3.4)$$

and for comparison we also have

$$S_{sem} S_s = s_{Pl} S_{QFT}, \quad \left( s_{Pl} = \frac{\pi k_B}{4} \right) \quad (3.5)$$

where

$$S_{QFT} = \frac{\pi k_B}{4} \left( \frac{L}{l_{Pl}} \right)^3 \quad \text{and} \quad S_s = \frac{\pi k_B}{4} \left( \frac{L}{l_s} \right) \quad (3.6)$$

In pure QFT (without gravity) the number of modes of the fields is proportional to the volume of the system (ie a box), and a short distance external cut-off is necessary since QFT ultraviolet divergences, naturally placed here by quantum gravity scale  $l_{Pl}$ . The string entropy  $S_s$  is proportional to the length. Clearly, the semiclassical gravity entropy  $S_{sem}$  “interpolates” between the pure (without gravity) QFT entropy  $S_{QFT}$  and the (quantum gravity) string entropy  $S_s$ . Expressions (3.3)-(3.5) for  $S_{sem}$  explicitly exhibit its semiclassical nature, ie,  $L_{cl} \gg l_{Pl}$  (equivalently,  $M_{cl} \gg m_{Pl}$ ,  $\mathcal{K}_{cl} \ll \kappa_{Pl}$ ,  $T_{sem} \ll t_{Pl}$ ). The expression

$$S_{sem} = \frac{k_B}{l_{Pl}^{D-2}} \frac{A_{BH}}{4} \quad (3.7)$$

holds in any spacetime dimensions with the black hole area  $A_{BH} = A_{D-2} R_{BH}^{D-2}$  ( $A_{D-2}$  and  $R_{BH}$  being given by eq (2.11)). In terms of  $L_{cl}, M_{cl}$ , it reads :

$$S_{sem} = \frac{k_B}{4} A_{D-2} \left( \frac{(D-3)L_{cl}}{2l_{Pl}} \right)^{D-2} = \frac{\pi k_B}{4} \left( \frac{L_{clD}}{l_{Pl}} \right)^2 = \frac{\pi k_B}{4} \left( \frac{M_{clD}}{m_{Pl}} \right)^2 \quad (3.8)$$

with

$$L_{clD} = \sqrt{\frac{A_{D-2}}{\pi}} \left( \frac{16\pi G M_{BH}}{c^2(D-2)A_{D-2}} \right)^{\frac{1}{2}(\frac{D-2}{D-3})}, \quad M_{clD} \equiv \frac{c^2}{G} L_{clD} \quad (3.9)$$

(for  $D=4$ :  $L_{cl4} = L_{cl}$ ,  $M_{cl4} = M_{cl}$ ). Or,

$$S_{sem} = \frac{1}{8} \left( \frac{M_{clD} c^2}{T_{sem}} \right) = \frac{\pi k_B}{4} \frac{T_{cl}(M_{clD})}{T_{sem}} \quad (3.10)$$

$$T_{cl} = \frac{1}{2\pi k_B} M_{clD} c^2, \quad T_{sem} = \frac{1}{2\pi k_B} M_{semD} c^2 \quad (3.11)$$

In the quantum gravity regime, black hole mass is quantized, several arguments in the context of QFT (horizon area quantization, canonical quantization) lead to the condition [16],[17],[18] :

$$M_{cl}^2 = \frac{\hbar c}{G} N = m_{Pl}^2 N, \quad N = 0, 1, \dots \quad \text{ie} \quad M_{cl} = \left( \frac{2\pi k_B}{c^2} t_{Pl} \right) \sqrt{N} \quad (3.12)$$



On the other hand, the string mass spectrum and string density of states in black hole (asymptotically flat)[4] space times are the same as in flat space time, ie

$$M^2 = \frac{\hbar}{c\alpha'} n = m_s^2 n, \quad n = 0, 1, \dots \quad (3.13)$$

$$\rho_s(M) = e^{S_s/k_B} \sim e^{Mc^2/(8k_B T_s)} = e^{2\pi b\sqrt{n}} \quad (3.14)$$

$$S_s = \frac{\pi k_B}{4} \left( \frac{L}{L_s} \right) = \frac{\pi k_B}{4} \left( \frac{M}{M_s} \right) = \frac{1}{8} \frac{M c^2}{T_s} \quad (3.15)$$

$S_s$  and  $T_s$  being the string entropy and string temperature respectively,

$$S_s = 2\pi k_B b\sqrt{n}, \quad T_s = \frac{t_s}{8b} = \frac{1}{2\pi k_B} \frac{m_s c^2}{8b} \quad (3.16)$$

here

$$L_s = 8b l_s, \quad M_s = \frac{m_s}{8b} \quad (3.17)$$

ie,

$$M = \left( \frac{2\pi k_B}{c^2} t_s \right) \sqrt{n} = \left( \frac{2\pi k_B}{c^2} 8b T_s \right) \sqrt{n} \quad (3.18)$$

$l_s$ ,  $m_s$  being the fundamental string length and mass respectively eq (2.20). The numerical coefficient  $b$  depends on  $D$  and on the type of strings, for instance :

$$b = 2\sqrt{\frac{D-2}{6}} \quad \text{for bosonic strings, (closed and open).} \quad (3.19)$$

Useful expressions for  $T_s$  and  $S_s$ , (to compare to the black hole) are :

$$T_s = \frac{\hbar c}{2\pi k_B} \frac{1}{L_s} = \frac{1}{2\pi k_B} M_s c^2 = \frac{c}{2\pi k_B} \frac{1}{8b} \sqrt{\frac{\hbar c}{\alpha'}} \quad (3.20)$$

$$S_s = \frac{\pi k_B}{4} \frac{T(M)}{T_s}, \quad T(M) \equiv \frac{1}{2\pi k_B} M c^2 \quad (3.21)$$

The black hole density of states and entropy ( $\rho_{sem}$ ,  $S_{sem}$ ) Eqs (3.1)-(3.3) are the semiclassical expressions of the string expressions ( $\rho_s$ ,  $S_s$ ) eqs (3.14)-(3.15). In the quantum string regime,  $T_{sem}$  becomes  $T_s$ , [2], [3]. [Eqs (3.1)-(3.4)] and [Eqs (3.14)-(3.15)] are the same physical quantities in different energy regimes.  $\rho_{sem}$  and  $\rho_s$  are given by eqs (3.1) and (3.14) respectively (at the leading behaviour). The black hole mass spectrum eq (3.12) becomes the string mass spectrum eq (3.18). We analyze it below.

## B. Quantum black hole spectrum

Eq (3.14) is the leading term to  $S_s$  for large  $M$  (large  $n$ ). The asymptotic behaviour of the string degeneracy  $d_n$  is

$$d_n \simeq (2\pi b\sqrt{n})^{-a} e^{2\pi b\sqrt{n}} = e^{S_s/k_B} \quad (3.22)$$

Or, in terms of  $M$

$$e^{S_s/k_B} = \rho_s(M) \simeq \left( \frac{1}{8k_B} \frac{M c^2}{T_s} \right)^{-a} e^{\frac{1}{8k_B} \frac{M c^2}{T_s}} \quad (3.23)$$

where for bosonic strings

$$a = D \quad (\text{closed strings}), \quad a = (D - 1)/2 \quad (\text{open strings}). \quad (3.24)$$

From eq (3.23) and eqs (3.1)-(3.3) we are able to write the corrections to the black hole entropy  $S_{sem}$ , such that in the string limit  $T_{sem} \rightarrow T_s$ ,  $(\rho_{sem}, S_{sem})$  maps into  $(\rho_s, S_s)$  :

$$e^{S_{sem}/k_B} = \rho_{sem}(M) \simeq \left( \frac{1}{8k_B} \frac{M_{cl}c^2}{T_{sem}} \right)^{-a} e^{\frac{1}{8k_B} \frac{M_{cl}c^2}{T_{sem}}} \quad (3.25)$$

ie,

$$S_{sem} = \frac{1}{8} \frac{M_{cl}c^2}{T_{sem}} - ak_B \log \left( \frac{1}{8k_B} \frac{M_{cl}c^2}{T_{sem}} \right) \quad (3.26)$$

Or, from eq (3.3), (which we stand by  $S_{sem}^{(0)}$ , the leading term) :

$$S_{sem} = S_{sem}^{(0)} - ak_B \log \left( \frac{S_{sem}^{(0)}}{k_B} \right) = \frac{\pi k_B}{4} \left( \frac{L_{cl}}{l_{Pl}} \right)^2 - ak_B \log \left[ \frac{\pi}{4} \left( \frac{L_{cl}}{l_{Pl}} \right)^2 \right] \quad (3.27)$$

This fix the logarithmic correction and its coefficient. Corrections to the semiclassical BH entropy have been continuously treated in the litterature, in the context of QFT [38], [39],[40] or strings (using correspondences, principles, “stretched horizon”, “Rindler wedge”, etc) [14], [34]. We do not make use here of such assumptions or identifications. Eqs (3.25) and (3.23) leads us to the black hole quantization condition for the highly excited levels :

$$\frac{1}{8k_B} \frac{M_{cl}c^2}{T_{sem}} = 2\pi 8b^2 N, \quad \text{high } N \quad (3.28)$$

$$\text{ie,} \quad M_{cl} = \left( \frac{2\pi k_B}{c^2} (8b)^2 T_{sem} \right) N \quad (3.29)$$

This condition is such that in the quantum gravity regime, the black hole mass spectrum becomes the string mass spectrum. Eqs (3.28), (3.29) yields the black hole spectrum :

$$S_{sem} = 2\pi k_B 8b^2 N - ak_B \log(2\pi 8b^2 N) \quad (3.30)$$

$$M_{cl} = m_{Pl} (8b\sqrt{N}), \quad M_{sem} = \frac{m_{Pl}}{(8b\sqrt{N})}, \quad T_{sem} = \frac{t_{Pl}}{(8b\sqrt{N})} \quad (3.31)$$

It displays the classical-quantum gravity dual nature eq(2.13),  $M_{sem} M_{cl} = m_{Pl}^2$ ,  $T_{sem} T_{cl} = t_{Pl}^2$ . In any number of dimensions, the black hole mass spectrum eq (3.29) for  $M_{cl}$  is like the flat space string spectrum eq (3.18) (for a string with mass  $m_s = 8bm_{Pl}$ ).  $M_{cl}$  is connected to the black hole radius  $R_{BH}$  and to the black hole mass  $M_{BH}$  as given by eqs (2.11)-(2.12), we have then for  $R_{BH}$  and  $M_{BH}$  :

$$R_{BH} = \frac{(D-3)}{2} l_{Pl} (8b\sqrt{N}), \quad M_{BH} = \mathcal{A}_D \left[ \frac{(D-3)}{2} l_{Pl} (8b\sqrt{N}) \right]^{D-3}, \quad \mathcal{A}_D \equiv \frac{c^2(D-2)}{16\pi G} A_{D-2} \quad (3.32)$$

This is the quantized black hole radius and black hole mass in any number of dimensions.

### C. Unified quantum decay of QFT elementary particles, black holes and strings

An unified description for the quantum decay rate of unstable heavy particles is provided by the formula[19]

$$\Gamma = \frac{g^2 m}{\text{numerical factor}} \quad (3.33)$$

where  $g$  is the (dimensionless) coupling constant,  $m$  is the typical mass in the theory (the mass of the unstable particle object) and the numerical factor often contains the relevant mass ratios in the decay process. This formula nicely encompass all the particle width decays in the standard model (muons, Higgs, etc), as well as the decay width of topological and non topological solitons, cosmic defects and fundamental quantum strings [19].

A quantum closed string in an  $N$ th excited state decays into lower excited states (including the dilaton, graviton and massless antisymmetric tensor fields) with a total width (to the dominant order (one string loop)) given by[20]

$$\Gamma_s = \frac{G}{n^2} T_s^3 \sim \frac{G}{l_s^3} \quad (3.34)$$

which can be also written

$$\Gamma_s = \frac{g^2}{n^2} m_s, \quad (3.35)$$

( $n^2$  is a numerical factor). That is, the string decay  $\Gamma_s$  eq (3.35) has the same structure as eq (3.33) with  $g \equiv \sqrt{\frac{G}{\alpha'}}$ .

On the other hand, a semiclassical black hole decays thermally as a “grey body” at the Hawking temperature  $T_{sem}$  (the “grey body” factor being the black hole absorption cross section  $\sim L_{cl}^2$ ). The mass loss rate (estimated from a Stefan-Boltzmann relation) is

$$\left( \frac{dM_{cl}}{dt} \right) = -\sigma L_{cl}^2 T_{sem}^4 \sim T_{sem}^2 \quad (3.36)$$

where  $\sigma$  is a constant. Then, the semiclassical black hole decay rate is given by

$$\Gamma_{sem} = \left| \frac{d}{dt} \ln M_{cl} \right| \sim \frac{G}{n^2} T_{sem}^3 \sim \frac{G}{L_{cl}^3} \quad (3.37)$$

As evaporation proceeds, the black hole temperature increases until it reaches the string temperature [2], the black hole enters its string regime  $T_{sem} \rightarrow T_s$ ,  $L_{cl} \rightarrow L_s$ , becomes a string state, then decays with a width

$$\Gamma_{sem} \rightarrow G T_s^3 \sim \frac{G}{l_s^3} \rightarrow \Gamma_s \quad (3.38)$$

The semiclassical black hole decay rate  $\Gamma_{sem}$  tends to the string decay rate  $\Gamma_s$ . Eq (3.27) is the semiclassical expression of eq (3.38). Again, unification between black hole decay and elementary particle decay is achieved for quantum black holes, when the black hole enters its quantum gravity (string) regime.

#### IV. SEMICLASSICAL (QFT) AND QUANTUM (STRING) DE SITTER STATES

The classical and semiclassical set of quantities ( $L_{cl}$ ,  $\mathcal{K}_{cl}$ ,  $M_{cl}$ ,  $T_{sem}$ ) corresponding to QFT in de Sitter background [21],[22], and the string set of quantities ( $L_s$ ,  $\mathcal{K}_s$ ,  $M_s$ ,  $T_s$ ) derived from quantum string dynamics in de Sitter background [23],[5], [6], [2](canonical as well as semiclassical quantization) are the following :

$$\text{QFT : } L_{cl} = cH^{-1}, \quad \mathcal{K}_{cl} = cH, \quad M_{cl} = \frac{c^3}{GH} \quad (4.1)$$

$$T_{sem} = \frac{\hbar}{2\pi k_B} \mathcal{K}_{cl} = \frac{\hbar}{2\pi k_B} H = \frac{\hbar c}{2\pi k_B} L_{cl}^{-1} \quad (4.2)$$

$$T_{sem} = \frac{1}{2\pi k_B} M_{sem} c^2, \quad M_{sem} = \frac{m_{Pl}^2}{M_{cl}} = \frac{\hbar}{c^2} H \quad (4.3)$$

$$\text{String : } L_q \equiv L_s = \frac{\alpha' \hbar}{c^2} H = \frac{\alpha' \hbar}{c L_{cl}} \quad (4.4)$$

$$\mathcal{K}_s = \frac{c^2}{L_s} = \frac{c^4}{\hbar \alpha' H} = \frac{c^3}{\hbar \alpha'} L_{cl} \quad (4.5)$$

$$M_s = \left( \frac{c}{\alpha' H} \right) = \frac{\hbar}{c L_s} = \frac{1}{\alpha'} L_{cl} \quad (4.6)$$

$$T_s = \frac{\hbar}{2\pi k_B c} \mathcal{K}_s = \frac{c^3}{2\pi k_B} \frac{1}{\alpha' H} = \frac{\hbar c}{2\pi k_B} \frac{1}{L_s} = \frac{1}{2\pi k_B} M_s c^2 \quad (4.7)$$

These expressions hold in any number of space-time dimensions (D): (D enters only through the relation between H and the scalar curvature, or cosmological constant  $\Lambda$ ):

$$R = D(D-1) \frac{H^2}{c}, \quad H = c \sqrt{\frac{2\Lambda}{(D-1)(D-2)}} \quad (4.8)$$

In the string quantities, D and the string model enter only through a numerical coefficient in  $L_s$ , as in flat space-time, (recall  $L_{s \text{ flat}} = \frac{8b(D-3)}{4\pi} l_s$ , the numerical coefficient b depending on the string type (closed, open, supersymmetric, etc)).

As can be seen, the classical/semiclassical (QFT) set  $(L_{cl}, M_{sem}, \mathcal{K}_{cl}, T_{sem})$  eqs (4.1)-(4.3) and the quantum string set  $(L_s, M_s, \mathcal{K}_s, T_s)$  eqs (4.4)-(4.7) satisfy the classical-quantum relations, eqs (2.21)-(2.22). The set  $(L_s, M_s, \mathcal{K}_s, T_s)$  is the **quantum string dual** (in the sense of the wave-particle-string duality) of the classical/QFT set  $(L_{cl}, M_{sem}, \mathcal{K}_{cl}, T_{sem})$  :

$$L_s = l_s^2 L_{cl}^{-1}, \quad M_s = m_s^2 M_{sem}^{-1}, \quad \mathcal{K}_s = \kappa_s^2 \mathcal{K}_{cl}^{-1}, \quad T_s = t_s^2 T_{sem}^{-1} \quad (4.9)$$

$L_{cl}$ ,  $\mathcal{K}_{cl}$  and  $M_{cl}$  are the radius, acceleration (surface gravity) and “mass” of the de Sitter (dS) universe respectively.  $T_{sem}$  is the Hawking-Gibbons temperature: the QFT temperature in dS background, or the intrinsic dS temperature in its semiclassical(QFT) regime.  $L_s$ ,  $\mathcal{K}_s$  and  $M_s$  are the size, acceleration and mass of quantum strings in dS universe.  $T_s$  is the string temperature (the maximal temperature of strings) in dS background [1],[3], which is also the intrinsic temperature of dS universe in the string regime : in the string analogue model with back reaction included, strings self-sustain a de Sitter phase of high curvature with temperature  $T_s$  as given by eq (51) , [1] and sub- sections below.

### A. Density of states and entropy

The entropy and density of states of semiclassical (QFT) de Sitter background are [21],[24]

$$\rho_{sem} = e^{S_{sem}/k_B} \quad (4.10)$$

$$S_{sem} = \frac{k_B c^3}{G \hbar} \frac{A_{cl}}{4} = \pi k_B \left( \frac{L_{cl}}{l_{Pl}} \right)^2 = \frac{\pi k_B}{l_{Pl}^2} \left( \frac{c}{H} \right)^2 \quad (4.11)$$

Or,

$$S_{sem} = \pi k_B \left( \frac{M_{cl}}{m_{Pl}} \right)^2 = \frac{1}{2} \frac{M_{cl} c^2}{T_{sem}} = \pi k_B \frac{T_{cl}}{T_{sem}} \quad (4.12)$$

On the other hand, the mass formula for quantum strings in the de Sitter background [23],[5],[6] is given by :

$$\left( \frac{m}{m_s} \right)^2 = 24 \sum_{n>0} \frac{n^2 + \omega_n^2(m)}{\omega_n(m)} + 2\mathcal{N} \frac{1 + \omega_1^2(m)}{\omega_1(m)} \quad (4.13)$$

$$\text{where } \omega_n(m) = \sqrt{n^2 - \left(\frac{m}{M_s}\right)^2} \quad \text{and } \mathcal{N} \text{ is the number operator} \quad (4.14)$$

$$\mathcal{N} = \frac{1}{2} \sum_{n>0} n [\alpha_n^{R+} \alpha_n^R + \tilde{\alpha}_n^{R+} \tilde{\alpha}_n^R], \quad [\alpha_n, \alpha_n^+] = 1 = [\tilde{\alpha}_n, \tilde{\alpha}_n^+] \quad (4.15)$$

For large n :

$$\left(\frac{m}{m_s}\right)^2 \simeq 4n \left[1 - n \left(\frac{m_s}{M_s}\right)^2\right], \quad \text{with} \quad \left(\frac{m_s}{M_s}\right)^2 = \frac{\alpha' \hbar}{c} \left(\frac{H}{c}\right)^2 = \left(\frac{l_s}{L_{cl}}\right)^2 = \frac{L_s}{L_{cl}} \quad (4.16)$$

The degeneracy  $d_n(n)$  of level n (counting of oscillator states) is the same in flat as well as in curved space-time. The differences due to the background curvature enter through the relation  $m = m(n)$  of the mass spectrum. The density  $\rho(m)$  and the degeneracy  $d_n$  satisfy the relation  $\rho(m)dm = d_n(m)dn$ . From this identity and eq.(4.13), we find the string mass density of states in de Sitter space as given by

$$\rho_s(m) = f \left(\frac{m}{M_s}\right) e^{2\pi b \frac{M_s}{m_s} \sqrt{1 - \left(\frac{m}{M_s}\right)^2}} \quad (4.17)$$

where

$$f \left(\frac{m}{M_s}\right) = \frac{m/m_s}{\sqrt{1 - \left(\frac{m}{M_s}\right)^2}} \frac{(m_s/M_s)^4}{\left(1 - \sqrt{1 - \left(\frac{m}{M_s}\right)^2}\right)^2} \quad (4.18)$$

and  $M_s$  is given by eq (4.6). For low ( $m \ll M_s$ ) and high ( $m \rightarrow M_s$ ) masses, we have respectively :

$$\rho_s(m)_{m \ll M_s} \sim \left(\frac{m}{m_s}\right)^{-a} e^{2\pi b \sqrt{\frac{\alpha' \hbar}{c}} m \left[1 - \frac{1}{8} \left(\frac{\alpha' H}{c}\right)^2 m^2 + O\left(\frac{m}{M_s}\right)^3\right]} \quad (4.19)$$

$$\rho_s(m)_{m \rightarrow M_s} \sim \left(\frac{M_s}{m}\right)^{-a} \left(\frac{M_s}{\Delta m}\right)^{1/2} e^{2\pi b \frac{c}{H} \sqrt{\frac{c}{\alpha' \hbar}} \left[1 - \left(\frac{\Delta m}{2M_s}\right)^{1/2} - \frac{1}{2} \left(\frac{\Delta m}{M_s}\right) + O\left(\frac{\Delta m}{M_s}\right)^{3/2}\right]} \quad (4.20)$$

where  $a=D$  (closed strings) and

$$\Delta m \equiv (M_s - m_s), \quad M_s = \frac{c}{\alpha' H} \quad (4.21)$$

That is, (leading behaviour) :

$$\rho_s(m)_{m \ll M_s} \sim \left(\frac{m}{m_s}\right)^{-a} e^{2\pi b m/m_s} \quad (4.22)$$

$$\rho_s(m)_{m \rightarrow M_s} \sim \left(\frac{M_s}{m_s}\right)^{-a} \left(\frac{M_s}{\Delta m}\right)^{1/2} e^{2\pi b M_s/m_s} \quad (4.23)$$

Or, in terms of temperature :

$$\rho(T \ll T_s) \simeq \left(\frac{T}{t_s}\right)^{-a} e^{T/t_s} \quad (4.24)$$

$$\rho(T \rightarrow T_s) \simeq \left(\frac{T_s}{t_s}\right)^{-a} \sqrt{\frac{T_s}{(T - T_s)}} e^{T_s/t_s} \quad (4.25)$$

$$T \equiv \frac{1}{2\pi k_B} mc^2, \quad T_s \equiv \frac{1}{2\pi k_B} M_s c^2 = \frac{c^2}{2\pi k_B} (\alpha' H)^{-1} \quad (4.26)$$

For low masses, the string spectrum in dS background is like the flat space string spectrum [1],[5],[6]. For high masses, the spectrum is saturated by the maximal mass  $c/\alpha' H$  for the oscillating (particle) states. The density  $\rho_s(m)$  eqs (4.22)-(4.23), similarly expresses in terms of the typical mass scale in each domain:  $m_s$  (as in flat space) for low masses,  $M_s$  for high masses corresponding to the maximal number of oscillating states :

$$N_{max} \sim \text{Int} \left[ \frac{L_{cl}}{L_s} \right] \sim \text{Int} \left[ \frac{c^3}{\hbar \alpha' H^2} \right]. \quad (4.27)$$

(de Sitter space-time being bounded, this number although large is *finite*, and so arbitrary high masses can not be supported [5],[6],[1],[23]). When  $m > M_s$ , the string does not oscillate (it inflates with the background, the proper string size is larger than the horizon [25]), thus the string becomes “*classical*” reflecting the classical properties of the background. When  $m \rightarrow M_s$ , the string mass becomes the mass  $M_{cl}$  of de Sitter background eq.(4.1), (with  $\alpha'$  instead of  $G/c^2$ ), and conversely,  $M_s$  is also the mass of de Sitter background in its string regime,  $M_s \Leftrightarrow (M_{cl})$ . String back reaction in the string analogue model supports this fact : a de Sitter phase having mass  $M_s$  and temperature  $T_s$  given by eqs (4.6) and (4.7) respectively, is sustained by strings [1]. Therefore, we interpret  $(L_s, K_s, M_s, T_s)$  given by eqs (4.4)-(4.7) as the *intrinsic* size, surface gravity, mass and temperature of de Sitter background in its string (high energy) regime. When the string mass becomes  $M_s$ , it saturates de Sitter universe (the string size  $L_s$  (or Compton length) for this mass becomes the horizon size (ie,  $L_s \equiv L_q = \frac{c}{H}$ ), that is the string becomes “*classical*”, (or the background becomes quantum). The highly excited string states are very *quantum* as elementary particle states, but are “*classical*” as gravity states. The wave-particle-string duality of quantum gravity reflects for the same states the classical gravity properties (with  $\alpha'$  playing the role of  $G/c^2$ ), and the highly quantum properties (with  $L_s$  being the Compton length).

De Sitter background is an exact solution of the semiclassical Einstein equations with the QFT back reaction of matter fields included [22],[26]. dS background is also a solution of the semiclassical Einstein equations with the string back reaction (in the string analogue model) included [1]: the curvature is a function of  $(T_{sem}, T_s)$  and contains the QFT semiclassical curvature as a particular case (for  $T_{sem} \ll T_s$ ). The QFT semiclassical de Sitter background is a low energy phase (for  $T_{sem} < T_s$ ,  $\mathcal{K}_{cl} < \mathcal{K}_s$ ). When  $T_{sem} \rightarrow T_s$ , then  $\mathcal{K}_{cl} \rightarrow \mathcal{K}_s$  and from eqs (4.1) (4.4), (4.10) and (4.17) it follows that

$$L_{cl} \rightarrow L_s, \quad \text{ie} \quad \frac{c}{H} \rightarrow l_s, \quad (4.28)$$

the QFT semiclassical de Sitter phase becomes a string state selfsustained by a string cosmological constant

$$\Lambda_s = \frac{1}{2l_s^2} (D-1)(D-2), \quad (4.29)$$

this string state only depends on  $\alpha'$  (and  $\hbar, c$ ). Size, surface gravity and temperature of the de Sitter string phase are

$$H_s = \frac{c}{l_s}, \quad \mathcal{K}_s = \frac{c^2}{l_s}, \quad T_s = \frac{\hbar c}{2\pi k_B} \frac{1}{l_s}, \quad (4.30)$$

corresponding to a de Sitter maximal curvature

$$R_s = D(D-1) \frac{c}{l_s^2} \quad (4.31)$$

This is also supported by the string back reaction computation [1]: the leading term of the de Sitter curvature in the quantum regime is given by eq (4.31) plus negative corrections in an expansion in powers of  $(R_{sem}/R_s)$ ,  $R_{sem}$  being the semiclassical (QFT) de Sitter curvature. The two phases: semiclassical and stringy are dual of each other (in the sense of the classical/quantum gravity duality) satisfying eqs (2.21)-(2.23).

## B. Quantum de Sitter spectrum

From the asymptotic degeneracy of states  $d_N$  and eq (56) we can write the corrections to the semiclassical entropy of de Sitter space :

$$S_{sem} = S_{sem}^{(0)} - ak_B \log \left( \frac{S_{sem}^{(0)}}{k_B} \right), \quad S_{sem}^{(0)} = \frac{\pi k_B}{l_{Pl}} \left( \frac{c}{H} \right)^2 \quad (4.32)$$

Similarly to the BH case, this gives us the quantization condition of de Sitter background

$$S_{sem} = 2\pi k_B 2b^2 N - ak_B \log(2\pi 2b^2 N) \quad (4.33)$$

$$M_{cl} = \left( \frac{2\pi k_B}{c^2} T_{sem} \right) (2b)^2 N, \quad b = \sqrt{\frac{D-2}{6}} \quad (4.34)$$

$$\text{that is, } M_{cl} = m_{Pl} 2b\sqrt{N}, \quad L_{cl} = l_{Pl} 2b\sqrt{N} \quad (4.35)$$

Or, for  $H$ ,  $R$  and  $\Lambda$  :

$$H_N = \left( \frac{c}{l_{Pl}} \right) \frac{1}{(2b\sqrt{N})}, \quad R_N = D(D-1) \frac{c}{l_{Pl}^2} \frac{1}{(2b\sqrt{N})^2}, \quad \Lambda_N = \frac{3}{4l_{Pl}^2} \frac{(D-1)(D-2)}{N} \quad (4.36)$$

The quantum mass de Sitter spectrum eq (4.35) is like the highly excited string spectrum eq (4.16), with  $m_s = m_{Pl}b$ . As  $(G/c^2 \rightarrow \alpha')$  this spectra are the same. For increasing  $N$  the string levels in dS tend to mix up and pile up towards  $N_{max}$ , which is symptomatic feature of a phase transition at the  $M_s$  (or  $T_s$ ) string temperature in de Sitter background eq (4.26). This phase transition is confirmed from the computation of the string density of states and entropy : from eq.(4.17) the string in de Sitter background has an entropy  $S_s = k_B \log \rho_s$  equal to

$$S_s = \frac{\pi k_B}{4} \left[ 1 - \sqrt{1 - \left( \frac{m}{M_s} \right)^2} \right]^{1/2} + k_B \log f \left( \frac{m}{M_s} \right) \quad (4.37)$$

where  $f(m/M_s)$  is given by eq (4.18). For low ( $m \ll M_s$ ) and high ( $m \rightarrow M_s$ ) masses we have

$$S_s(m \ll M_s) = k_B \left( \frac{m}{m_s} \right) - ak_B \log \left( \frac{m}{m_s} \right) \quad (4.38)$$

$$S_s(m \rightarrow M_s) = k_B \left( \frac{M_s}{m_s} \right) - ak_B \log \left( \frac{M_s}{m_s} \right) + k_B \log \sqrt{\frac{M_s}{m - M_s}} \quad (4.39)$$

Or, in terms of temperature eqs(4.26) :

$$S_s(T \ll T_s) = k_B \left( \frac{T}{T_s} \right) - ak_B \log \left( \frac{T}{T_s} \right) \quad (4.40)$$

$$S_s(T \rightarrow T_s) = k_B \left( \frac{T_s}{t_s} \right) - ak_B \log \left( \frac{T_s}{t_s} \right) + k_B \log \sqrt{\frac{T_s}{T - T_s}} \quad (4.41)$$

The term  $\log \sqrt{\frac{M_s}{m - M_s}} = \log \sqrt{\frac{T_s}{T - T_s}}$  indicates the presence of a phase transition at the string de Sitter temperature  $T_s = \frac{c^2}{2\pi k_B} (\alpha' H)^{-1}$ . Remarkably, the quantum mass spectrum, density of states and entropy of strings in de Sitter background account for the quantization (quantum mass spectrum) of the semiclassical de Sitter background, density of states and entropy and in the string regime a phase transition takes place. The characteristic behaviour associated to this phase transition is the logarithmic singularity at  $T_s$  (or pole singularity in the specific heat) which is the same in all dimensions. The square root branch point at  $T_s$  is the same in all space-time dimensions. The statistical mechanics of self-gravitating non relativistic systems also shows a square root behaviour in the physical magnitudes near the critical point (found by mean field and Monte Carlo computations)[8]. This gravitational phase transition, due to the long range attractive gravitational interaction at finite temperature, is linked, although with more complex structure, to the Jeans'like instability, here at the string de Sitter temperature  $T_s$  eq (4.26).

## V. SEMICLASSICAL (QFT) AND QUANTUM (STRING) ANTI-DE SITTER STATES

AdS background alone has no event horizon and the surface gravity is zero. The Hawking temperature is zero in AdS, ie the AdS QFT vacuum has no intrinsic temperature,

$$T_{sem\,AdS} = 0 \quad (5.1)$$

On the other hand, there is no maximal temperature for strings in AdS,[5]. The mass spectrum of quantum strings does not have any maximal or critical mass and the partition function for a gas of strings in AdS is defined for *all* temperature [5],[6],

$$T_{s\,AdS} = \infty \quad (5.2)$$

Again, we see that QFT and quantum strings satisfy the dual relations eqs (2.21)-(2.22). The (H, R,  $\Lambda$ ) relation in AdS is the same as in de-Sitter space, (but R and  $\Lambda$  being negative) :

$$R = -D(D-1) \frac{H^2}{c}, \quad H = c \sqrt{\frac{2|\Lambda|}{(D-1)(D-2)}} \quad (5.3)$$

The quantities  $(L_{cl}, \mathcal{K}_{cl}, M_{cl}, M_{sem}, T_{sem})$  are the same as eqs(4.8) for de Sitter background. Although there is no horizon, these quantities set up typical classical and semiclassical scales for length, acceleration, mass and temperature in AdS background. Similarly,  $(L_s, \mathcal{K}_s, M_s, T_s)$  are typical length, acceleration, mass and temperature scales for strings in AdS. In particular,  $M_s$  is *not* a maximal mass for strings in AdS, but it sets up the typical AdS string mass scale : (i) low, (ii) intermediate and (iii) high string mass states in AdS correspond respectively to

$$(i) \, m \ll M_s, \quad (ii) \, m \sim M_s, \quad (iii) \, m \gg M_s, \quad \left( M_s = \frac{c}{\alpha' H} \right), \quad (5.4)$$

The regime (iii) is absent in de-Sitter background. The mass formula for strings in AdS spacetime is given by[5]

$$\left( \frac{m}{m_s} \right)^2 = 2 \sum_{n>0} \left( \frac{n^2 + \omega_n^2}{\omega_n} \right) + \sum_{n>0} \mathcal{N} \left( \frac{n^2 + \omega_n^2}{\omega_n} \right) \quad (5.5)$$

where  $\mathcal{N}$  is the number-operator eq (4.13) and  $\omega_n$  the frequency of the AdS string oscillators :  $\omega_n = \sqrt{n^2 + \left( \frac{m}{M_s} \right)^2}$ .

This is the same mass formula as in de Sitter background but with positive sign inside the square root in  $\omega_n$ , (instabilities *do not* appear in AdS alone, nor for strings nor for QFT). In the conformal invariant AdS background (WZWN model), the mass formula is very similar [7] :

$$\left( \frac{n}{m_s} \right)^2 = \left( \frac{m_+}{m_s} \right)^2 + \left( \frac{m_-}{m_s} \right)^2, \quad m_s = \sqrt{\frac{\hbar}{\alpha' c}} \quad (5.6)$$

with

$$\left( \frac{m_{\pm}}{m_s} \right)^2 = \sum_{n>0} \frac{(n^2 + \omega_{n\pm})}{\omega_{n\pm}} + \sum_{n>0} \mathcal{N}_{\pm} \frac{(n^2 + \omega_{n\pm}^2)}{\omega_{n\pm}}, \quad \omega_{n\pm} = \left| n \pm \frac{m}{M_s} \right|, \quad M_s = \frac{c}{\alpha' H} \quad (5.7)$$

The physics described by formulae (5.5) and (5.6) is the *same* : low and high mass states are *identical*, only intermediate mass states are slightly different, but these differences are minor. Low and high mass spectrum is given by

$$\left( \frac{m}{m_s} \right)^2 (m \ll M_s) = n \left[ 1 + O \left( \frac{m_s}{M_s} \right)^2 \right] \quad (5.8)$$

$$\left( \frac{m}{m_s} \right)^2 (m \gg M_s) = \left( \frac{m_s}{M_s} \right)^2 n^2 + n, \quad n \gg (M_s/m_s)^2 \quad (5.9)$$

ie

$$m = \frac{m_s^2}{M_s} n \left[ 1 + \left( \frac{M_s}{m_s} \right)^2 \frac{1}{n} \right]^{1/2}, \quad \frac{m_s^2}{M_s} = \frac{1}{\alpha'} \left( \frac{l_s^2}{L_{cl}} \right) = \frac{\hbar H}{c^2} \quad (5.10)$$

the high masses are independent of  $\alpha'$ , (the scale for the  $(m \gg M_s)$  states is sets up by H (and  $\hbar/c$ ))[5],[6],[7] and masses increase with n (not with  $\sqrt{n}$ ).



### A. String density of states and entropy in AdS.

Let us derive now the density of mass levels  $\rho_s(m)$ . From eq (5.5) and the identity  $\rho_s(m) dm = d_n(m) dn$ , we have

$$\rho_s(m) = \frac{2m}{m_s^2} \frac{d_n(m)}{\left[1 + 2n \left(\frac{m_s}{M_s}\right)^2\right]} \quad (5.11)$$

From  $d_n(n)$  eq (3.23) we find :

$$\rho_s(m) = \frac{2(m/m_s^2)}{\sqrt{1 + 4(m/M_s)^2}} (2\pi b \sqrt{n})^{-a} e^{2\pi b \sqrt{n}} \quad (5.12)$$

where

$$2\pi b \sqrt{n} = \sqrt{2}\pi b \left(\frac{M_s}{m_s}\right) \left[-1 + \sqrt{1 + 4\left(\frac{m}{M_s}\right)^2}\right]^{1/2}, \quad \frac{M_s}{m_s} = \frac{L_{cl}}{l_s} = \frac{c}{H} \sqrt{\frac{c}{\alpha' \hbar}} \quad (5.13)$$

For low, intermediate and high masses we have :

$$\rho_s(m \ll M_s) = 2 \left(\frac{m}{m_s^2}\right) \left(2\pi b \frac{m}{m_s}\right)^{-a} e^{2\pi b \frac{m}{m_s} \left[1 + \left(\frac{m}{M_s}\right)^2 + O\left(\frac{m}{M_s}\right)^4\right]} \left[1 - 2\left(\frac{m}{M_s}\right)^2\right] \quad (5.14)$$

$$\rho_s(m \sim M_s) \sim 2 \left(\frac{M_s}{m_s^2}\right) \left[2\pi b \left(\frac{M_s}{m_s}\right)\right]^{-a} e^{\sqrt{2}\pi b \left(\frac{M_s}{m_s}\right)} \quad (5.15)$$

$$\rho_s(m \gg M_s) = \left(\frac{M_s}{m_s^2}\right) \left(2\pi b \frac{\sqrt{m M_s}}{m_s}\right)^{-a} e^{2\pi b \frac{\sqrt{m M_s}}{m_s} \left[1 - \frac{1}{2}\left(\frac{M_s}{m}\right)\right]} \left[1 + O\left(\frac{M_s}{m}\right)\right] \quad (5.16)$$

Or, in terms of temperature :

$$\rho_s(T \ll T_s) = 2 \left(\frac{c^2}{2\pi k_B t_s}\right) \left(\frac{T}{T_s}\right) \left(2\pi b \frac{T}{t_s}\right)^{-a} e^{2\pi b T/t_s} \quad (5.17)$$

$$\rho_s(T \sim T_s) = 2 \left(\frac{c^2}{2\pi k_B t_s}\right) \left(\frac{T_s}{t_s}\right) \left(2\pi b \frac{T_s}{t_s}\right)^{-a} e^{2\pi b T_s/t_s} \quad (5.18)$$

$$\rho_s(T \gg T_s) = \left(\frac{c^2}{2\pi k_B t_s}\right) \left(\frac{T_s}{t_s}\right) \left(2\pi b \frac{\sqrt{T T_s}}{t_s}\right)^{-a} e^{2\pi b \frac{\sqrt{T T_s}}{t_s}} \quad (5.19)$$

Here we have only shown the dominant behaviours, the sub-leading terms can be read directly from eqs (5.13)-(5.14). The corresponding low, medium and high temperature behaviours of the entropy  $S_s = k_B \log \rho_s$ , are read from equations (5.17)-(5.18). We see that the characteristic mass ratio (or temperature) in  $\rho_s(m)$  and  $S_s$  goes from (i)  $m/m_s$  (or  $T/t_s$ ) for the low excited (or weak curvature) states as in flat space, to the new behaviour (iii)  $\sqrt{T T_s}/t_s$  for the highly excited states, (passing through the intermediate mass state ratio (ii)  $T_s/t_s$ ) :

$$(i) \frac{m}{m_s} (m \ll M_s) \longrightarrow (ii) \frac{M_s}{m_s} (m \simeq M_s) \longrightarrow (iii) \frac{\sqrt{m M_s}}{m_s} (m \gg M_s) \quad (5.20)$$

$$\text{ie, } (i) \frac{T}{t_s} (T \ll T_s) \longrightarrow (ii) \frac{T_s}{t_s} (T \simeq T_s) \longrightarrow (iii) \frac{\sqrt{T T_s}}{t_s} (T \gg T_s) \quad (5.21)$$

The characteristic low mass scale is the fundamental string mass  $m_s$ , while for the high masses, the scale is the AdS string mass  $M_s = c/\alpha'H$ . Interestingly enough, since

$$\frac{T_s}{t_s} = \frac{t_s}{T_{sem}}, \quad (5.22)$$

$\rho_s(m)$  can be also expressed in terms of the semiclassical (QFT) temperature scale  $T_{sem}$  :

$$\rho_s(TT_{sem} \ll t_s^2) = 2 \left( \frac{c^2}{2\pi k_B t_s} \right) \left( \frac{TT_{sem}}{t_s^2} \right) \left( 2\pi b \frac{T}{t_s} \right)^{-a} e^{2\pi b \frac{T}{t_s}} \quad (5.23)$$

$$\rho_s(TT_{sem} \sim t_s^2) = 2 \frac{c^2}{2\pi k_B T_{sem}} \left( 2\pi b \frac{t_s}{T_{sem}} \right)^{-a} e^{2\pi b \frac{t_s}{T_{sem}}} \quad (5.24)$$

$$\rho_s(TT_{sem} \gg t_s^2) = 2 \frac{c^2}{2\pi k_B T_{sem}} \left( 2\pi b \sqrt{\frac{T}{T_{sem}}} \right)^{-a} e^{2\pi b \sqrt{\frac{T}{T_{sem}}}} \quad (5.25)$$

$t_s(\alpha')$  present in the low excited regime (i) eq (5.23), *completely cancels out* in the highly excited regime (iii) eq (5.25). Eqs (5.17) or eqs (5.23), show that the low excited AdS string state (low values of  $\Lambda$ , low curvature), behaves like strings in flat space. For this state,  $\rho_s(m)$ , entropy, Hagedorn temperature take the flat space values, multiplied by corrections  $\left[ 1 + \sum_{n=1}^{\infty} c_n \left( \frac{\alpha' H^2}{c} \right)^n \right]$ . The highly excited AdS state (high values of  $\Lambda$ , high curvature) is *very different*

from flat space : the level spacing increases with  $n$ ;  $\rho_s(m)$ , entropy, etc, are functions of  $\sqrt{T/T_{sem}}$ , there is *no* maximal temperature, there is *no* finite (at finite temperature) critical point : it is pushed up to infinity by the negative  $\Lambda$ . There is no singularity at  $T \rightarrow T_s$  in  $\rho_s(m)$  nor in the entropy as in flat space or as in de Sitter space. In AdS,  $\rho_s(m)$ , the partition function, etc, are *all finite*, no phase transition occurs in AdS alone.

Notice that the so called Hawking-Page phase transition [9] which appears in black hole-AdS systems (BH-AdS) in the context of QFT is due to the black hole (not to AdS) : AdS space just sets up the boundary condition in asymptotic space such that the system BH-AdS be thermodynamically stable, and the semiclassical BH undergoes a transition at a critical temperature into a thermal gas. (BH's in dS space should also undergo a similar transition, although they are not, of course, thermally stable. Such phase transition should be of the same type of that found in the thermodynamics of self gravitating systems [8]).

## B. Quantum Anti-de Sitter spectrum

The role played by  $M_s$  (or  $T_s$  scale) in AdS is clear :  $M_s$  is the mass from which AdS differs drastically from flat space. For  $T \sim T_s$  ( $m \sim M_s$ ) or for  $T \gg T_s$  ( $m \gg M_s$ ), the string mass spectrum or  $\rho_s(m)$  in AdS gives us a quantization condition for  $M_s$  (or  $H$ ). Since

$$\frac{T_s}{t_s} = \frac{M_s}{m_s} = \frac{L_{cl}}{l_s} = \sqrt{\frac{c}{\alpha' \hbar}} \frac{c}{H} \quad (5.26)$$

from  $\rho_s(m)$  eq (5.18), we have at leading order

$$M_{sn} = m_s \sqrt{n}, \quad \text{ie} \quad H_n = \frac{c}{l_s} \frac{1}{\sqrt{n}}, \quad \Lambda_n = \frac{1}{2l_s^2} (D-1)(D-2) \frac{1}{n}, \quad (5.27)$$

which is the quantum spectrum for the background.

For  $T \gg T_s$ ,  $\rho_s(m)$  eq (5.19) leads

$$\sqrt{\frac{T}{T_{sem}}} = \sqrt{\frac{m}{M_{sem}}} = \sqrt{n}, \quad \text{ie} \quad m M_{cl} = m_{Pl}^2 n, \quad (5.28)$$

(which is the high  $n$  string spectrum in AdS). And when  $m \rightarrow M_{cl}$ , this yields

$$M_{cln} = m_{Pl} \sqrt{n}, \quad \text{ie} \quad H_n = \frac{c}{l_{Pl}} \frac{1}{\sqrt{n}}, \quad (5.29)$$

which is the quantization of  $M_{cl}$  (or  $H$ ) eq (5.27) (with  $l_{Pl}$  instead of  $l_s$ , ie  $G/c^2 \rightarrow \alpha'$ ). Again, when  $m \sim M_s$  and/or when  $m \gg M_s$ , the string becomes the background (and conversely, the high mass string spectrum accounts for the quantum spectrum of the background (the background becomes the string)).

## VI. CONCLUDING REMARKS

We have provided new results and understanding to the conceptual unification of the quantum properties of BH's, elementary particles, dS and AdS states. This is achieved by recognizing the relevant scales of the semiclassical (QFT) and quantum (string) regimes of gravity. They turn out to be the classical-quantum duals of each other, in the precise sense of the wave-particle (de Broglie, Compton) duality, here extended to the quantum gravity (Planck) domain: wave-particle-string duality.

Concepts as the Hawking temperature and string (Hagedorn) temperature are shown to be precisely the same concept in the different (semiclassical and quantum) gravity regimes respectively. Similarly, it holds for the Bekenstein-Hawking entropy and string entropy. An unifying formula for the density of mass states and entropy of BH, dS and AdS states has been provided in the two: semiclassical and string regimes. Quantum and string properties of the backgrounds themselves have been extracted. This is particularly enlightening for de Sitter background for several reasons:

- (i) the physical (cosmological) relevance of dS background (inflation at the early time, present acceleration);
- (ii) the lack at the present time of a full conformal invariant dS background in string theory;
- (iii) the results of string dynamics in conformal and non conformal string backgrounds which show that the physics remain mainly the same in the two class of backgrounds (conformal and non conformal) [5], [6], [7],[28];
- (iv) the phase transition found here for strings in dS background at the dS string temperature  $T_s$  eq(4.26), this temperature is the precise quantum dual of the semiclassical (QFT Hawking-Gibbons) dS temperature eq(4.2). This phase transition for strings in dS is the analogue of the Carlitz phase transition [29] for the thermodynamics of strings in flat space and in BH's [1].

A precise picture relating these string phase transitions among themselves and to the classical and semiclassical (QFT) gravitational phase transitions deserves future investigation but starts to be outlined here:

- (a) the string phase transitions at the string temperature  $T_s$  found in Minkowski [29], BH [1], and dS backgrounds [this paper] are all of the same nature (the value of  $T_s$  is different in each case).
- (b) This string phase transition does not occur in AdS alone (ie without BH,s), it could occurs in BH-AdS backgrounds.
- (c) These phase transitions are the string analogues of the so-called Hawking-Page transition [9] occurring in the semiclassical gravity regime (and which is due to the BH). Hawking-Page transition does not occurs in AdS alone (ie, without the BH).
- (d) All these phase transitions (for strings and QFT) should be the counterparts of the classical gravitational phase transitions occurring in the statistical mechanics of the selfgravitating gaz of particles [8] whose origin is (non linear) Jean's instability at finite temperature (usually called "gravothermal instability" [37], but with a richer structure: critical points, metastable phases, inhomogeneous ground state, fractal dimension [8]).

The description of the last stages of BH evaporation is not the central issue of this paper, but our results show that BH evaporation ends as quantum string decay into elementary particle states (most massless), ie pure (non mixed) quantum states.

The effects of BH angular momentum and charges can be included in this framework, as well as those of a cosmological constant to consider BH-dS, BH-AdS, etc, although we do not treat them here.

The exhibit of  $(\hbar, c, G)$  or  $(\hbar, c, \alpha')$  helps in recognizing the different relevant scales and regimes. Even if a hypothetical underlying "theory of everything" could only require pure numbers (option three in [30]), physical touch at some level asks for the use of fundamental constants [31],[30],[32],[33]. Here we favour *three* fundamental constants [31],[32],[33]  $((\hbar, c, G)$ , or  $(\hbar, c, \alpha')$ ), (tension being  $c^2/G$  or  $1/\alpha'$ ). (We do not use any further coupling, conceptual results here will not change by further couplings or interactions), (Hawking radiation, intrinsic gravitational entropy and other thermal features do not change by interactions or further background fields, dilaton, etc)

This paper does not make use of conjectures, proposals, principles, (CPP's) formulated in the last years in connection with string theory [34],[14]. String dynamics in curved backgrounds started well before CPP's,[23], many results for strings in curved backgrounds addressed nowadays (strings on AdS, on plane waves, semiclassical and canonical string quantization, rotating strings, null strings, integrable string equations, time dependent string backgrounds) can be found in [35], [36] and refs therein.

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